

Pairing instability near a lattice-influenced nematic quantum critical point

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We study how superconducting T_c is affected by a nematic quantum critical point (QCP) in a tetragonal environment. Including coupling of the electronic nematic variable to the relevant lattice strain restricts criticality only to certain high symmetry directions. This allows a weak-coupling treatment, even at the QCP. We develop a criteria distinguishing weak and strong T_c enhancements. We show that, if pairing is dominated by a non-nematic interaction away from the QCP, and if electron-strain coupling is sufficiently strong, then there is negligible enhancement of T_c upon approaching the QCP. We argue this is the case of the iron superconductors.

Introduction.— Understanding the origin of high superconducting transition temperature T_c of the copper and iron based systems continues to motivate a lot of theory and experimental research [1–5]. Among the various possibilities that are discussed as likely causes of high T_c , one is that of the presence of a quantum critical point (QCP) in the vicinity of which the effective pairing interaction is strong, leading to enhanced T_c . In fact, superconducting T_c boosted by the presence of an antiferromagnetic QCP remains among the more promising scenarios for these systems [6]. A related question is whether one expects similar enhancement of T_c close to a nematic QCP, where the ground state of the system is poised to break discrete rotational symmetry.

Several recent theoretical works have addressed this question, and have concluded that, indeed, the superconducting T_c is boosted near a nematic QCP [7–10]. The motivation for such a conclusion is intuitively clear, at least in the regime where weak-coupling theory of pairing is applicable. It is well-known that the effective electron-electron interaction mediated by the long-wavelength nematic fluctuations has a particular sign that, for small momentum transfer, leads to attractive interaction both in the s - and d -wave channels. This interaction increases as the system approaches a nematic QCP, leading to larger T_c . This analysis has been extended in Ref. [7] to Eliashberg type treatment to include the dynamics of the critical fluctuations, while Ref. [9] studied the fate of pairing when the underlying metal is a non Fermi liquid due to singularities generated by the quantum critical fluctuations in two dimensions. Both concluded that the intuitive picture stays in tact, and that approaching the nematic QCP leads to substantial enhancement of T_c .

Experimentally, at present there is no clear evidence of a nematic QCP in the phase diagram of the cuprates. For most iron based superconductors (FeSC) the nematic transition line and the QCP is well-identified [11–16], but the presence of a magnetic transition and its associated QCP complicates matter [5], since experimentally it is hard to disentangle the effects of the two. In this respect an ideal system is $\text{FeSe}_{1-x}\text{S}_x$, which has a nematic QCP but not a magnetic one. Interestingly, it is now well-

established that, in contradiction with the above theoretical expectations, in $\text{FeSe}_{1-x}\text{S}_x$ the superconducting T_c is hardly affected by the nematic QCP [17–19].

Note, the above theory works are based on electron-only models that ignore a symmetry-allowed coupling between the electronic nematic degree of freedom and a lattice shear strain mode. In this work we demonstrate the importance of this coupling in determining T_c enhancement near the QCP, which also allows us to understand qualitatively why T_c is unaffected by the nematic QCP in $\text{FeSe}_{1-x}\text{S}_x$.

It is well-known from the standard theory of elasticity that at an Ising-nematic acoustic instability, such as a second order tetragonal-orthorhombic transition, the divergence of the correlation length, which is manifested in vanishing acoustic phonon velocity, is restricted to certain high-symmetry directions [20–25]. Along the generic directions the non-critical shear strains, that are invariably present in a solid, come into play and cutoff criticality. This can be understood as follows.

Let us consider a translation symmetry preserving second order phase transition involving a local variable $\mathcal{X}(\mathbf{r}) = \mathcal{X} + \sum_{\mathbf{q} \neq 0} \mathcal{X}_{\mathbf{q}} e^{i\mathbf{q} \cdot \mathbf{r}}$, where \mathcal{X} is the order parameter. Within the Landau paradigm the free energy has mean field and fluctuation contributions. The former has the structure $F_{\text{MF}} = (a/2)\mathcal{X}^2 + (A/4)\mathcal{X}^4$, while the latter, to Gaussian order, is related to the action $S_{\text{fluc}} = \sum_{\mathbf{q} \neq 0} (b + q^2) |\mathcal{X}_{\mathbf{q}}|^2$. While in usual theories $b = a$, in those involving crystalline strains $b(\hat{q})$ is no longer a parameter but, rather, a function of the Brillouin zone angles \hat{q} containing information about the crystalline anisotropies [21, 22]. In other words, the concept of correlation length becomes angle-dependent. In this situation, the condition $b(\hat{q}) = a$ is satisfied only for certain high symmetry directions, and only along these directions the correlation length diverges at the transition defined by $a = 0$. Along other directions $b(\hat{q}) > 0$ at the transition, and correlation length stays finite.

The above property is inherited by the electronic nematic subsystem once its coupling with the strain is included in the critical theory [26–28]. This leads to two important conclusions concerning Cooper pairing, which

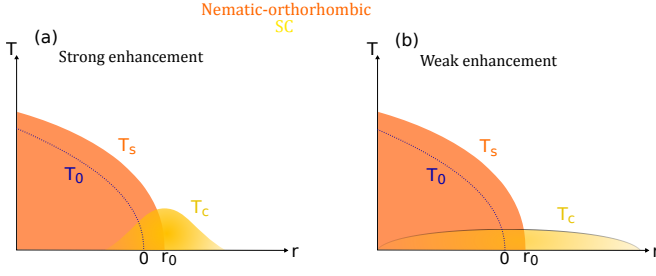


FIG. 1: (color online) Schematic phase diagrams with nematic/orthorhombic (orange) and superconducting (yellow) phases below temperatures $T_s(r)$ and $T_c(r)$, respectively. r is a dimensionless control parameter. $T_0(r)$ (dotted-lines) is the nominal electron-nematic transition in the absence of nemato-elastic coupling. The coupling shifts the nematic quantum critical point (QCP) from $r = 0$ to $r = r_0$. r_0 is a measure of the strength of the coupling (see text). (a) and (b) are two possible scenarios. In (a) there is “strong” enhancement of $T_c(r)$ at the QCP. In (b) the enhancement is “weak”, as in $\text{FeSe}_{1-x}\text{S}_x$ [17–19]. We show that (b) occurs if the pairing is dominated by a non-nematic potential away from the QCP, and if the nemato-elastic coupling is sufficiently strong (see Eq. (4)).

are the main results of this paper. (i) Under certain standard assumptions, the weak coupling BCS analysis is valid arbitrarily close to the nematic QCP. (ii) In the interesting situation where the pairing is dominated by a non-nematic interaction away from the QCP, significant enhancement of T_c is only possible when the electron-phonon interaction strength is sufficiently weak. In the opposite limit of strong coupling the enhancement is actually negligible (see Fig. 1). In this paper we identify the criteria that distinguishes between the two limits, and we argue that the FeSC belong to the latter limit.

Model.— We consider a system of itinerant electrons in a tetragonal lattice, with negligible dispersion along the z -axis, which is close to a nematic/structural QCP that is driven by electronic correlations. Ignoring electron-lattice interaction for the moment, the long wavelength fluctuations of the nematic variable $\phi_{\mathbf{q}}$, which is a collective mode of the electrons, is described by a susceptibility of the standard Ornstein-Zernike form $\chi_0^{-1}(\mathbf{q}) = r + q_{2d}^2/(2k_F)^2$, where $\mathbf{q}_{2d} \equiv (q_x, q_y)$, and r is a dimensionless tuning parameter of the theory that governs closeness to the QCP. Without the lattice coupling, the bare QCP is at $r = 0$. In what follows the frequency dependence of the susceptibility can be ignored.

A crucial ingredient in the model is the symmetry-allowed nemato-elastic term linking $\phi(\mathbf{r})$ with the local orthorhombic strain $\varepsilon(\mathbf{r}) = \varepsilon + i \sum_{\mathbf{q} \neq 0} [q_x u_x(\mathbf{q}) - q_y u_y(\mathbf{q})] e^{i\mathbf{q} \cdot \mathbf{r}}$, where $\vec{u}(\mathbf{r})$ is the atomic displacement, and ε is the uniform macroscopic strain, non-zero only in the symmetry-broken nematic/orthorhombic phase. This coupling can

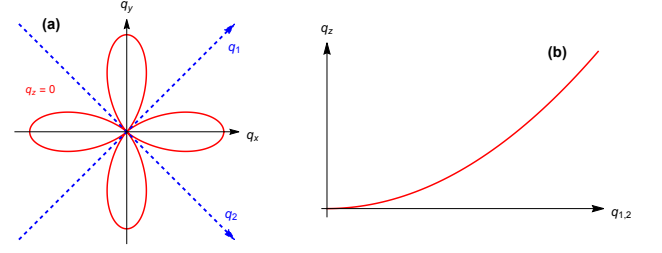


FIG. 2: (color online) The mass of the nematic susceptibility χ , defined as $r(\hat{q}) \equiv \lim_{\mathbf{q} \rightarrow 0} \chi^{-1}(\mathbf{q}, \omega = 0)$ becomes anisotropic in the presence of the nemato-elastic coupling. In the non-nematic phase the angular dependence of $r(\hat{q})$ has tetragonal symmetry. (a) Variation of $r(\hat{q})$ on the $q_z = 0$ plane (red line) at the nematic QCP. It is zero only along the high-symmetry directions $\hat{q}_{1,2} \equiv (\hat{q}_x \pm \hat{q}_y)/\sqrt{2}$ (blue arrows). (b) For finite q_z , $r(\hat{q}) \propto q_z^2$.

be written as $g \int d\mathbf{r} \phi(\mathbf{r}) \varepsilon(\mathbf{r})$, where g has dimension of energy.

The effect of the nemato-elastic term has been discussed earlier [28]. Here, for the sake of completeness, we recapitulate the main points. (i) It shifts the QCP to $r = r_0 \equiv g^2 \nu / C_0$ (see Fig. 1), where ν has dimension of density of states and C_0 is the bare orthorhombic elastic constant. Thus, r_0 is a dimensionless parameter that measures the strength of the nemato-elastic coupling. In the following we take $r_0 \leq r \leq 1$. (ii) The nemato-elastic coupling leads to hybridization of $\phi_{\mathbf{q}}$ with the acoustic phonons (see Fig. 3(a)), which renormalizes the nematic susceptibility to $\chi^{-1} = \chi_0^{-1} - \Pi$, with $\Pi(\hat{q}) = (g^2/\rho) \sum_{\mu} (\mathbf{a}_{\mathbf{q}} \cdot \hat{\mathbf{u}}_{\mathbf{q},\mu})^2 / \omega_{\mathbf{q},\mu}^2$. Here ρ is the density, μ is the polarization index, $\mathbf{a}_{\mathbf{q}} \equiv (q_x, -q_y, 0)$, and $\hat{\mathbf{u}}_{\mathbf{q},\mu}$ is the polarization vector for the bare acoustic phonons with angle-dependent velocity $\mathbf{v}_{\hat{q},\mu}^{(0)}$ and dispersion $\omega_{\mathbf{q},\mu} = \mathbf{v}_{\hat{q},\mu}^{(0)} \cdot \mathbf{q}$. Evidently, $\Pi(\hat{q})$ is independent of the magnitude q , and has four-fold symmetry of the tetragonal unit cell in the non-nematic phase. Thus, the nemato-elastic term makes the mass of $\phi_{\mathbf{q}}$ angle-dependent with $r \rightarrow r(\hat{q}) \equiv r + \Pi(\hat{q})$, and critically, or divergence of correlation length, is restricted to the high symmetry directions $\hat{q}_{1,2} \equiv (\hat{q}_x \pm \hat{q}_y)/\sqrt{2}$, for which $r(\hat{q}_{1,2}) = 0$ at the QCP (see Fig. 2). Along the remaining directions $r(\hat{q}) > 0$ at the QCP. Note, since divergence of χ also implies vanishing of the sound velocity *renormalized* by the coupling g , the above direction dependence is consistent with the fact that only along $\hat{q}_{1,2}$ the sound velocity vanishes at this nematic/structural transition [29, 30]. (iii) Since the coupling g cuts off divergence along the generic directions, the effect of the quantum fluctuations is weak. Therefore, below the temperature scale $T_{\text{FL}} \sim r_0^{3/2} T_F$ the system behaves as a Fermi liquid for thermodynamic and single-electron properties. Here T_F is the Fermi temperature. Above T_{FL} the nemato-elastic coupling can be

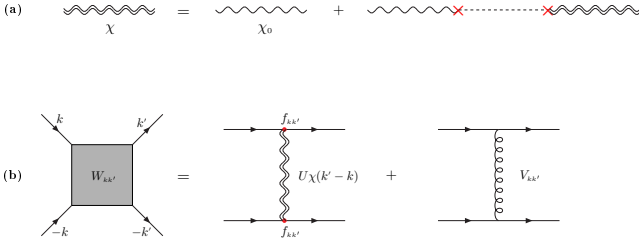


FIG. 3: (color online) Diagrammatic representation of the relevant microscopic processes. (a) The bare electron-nematic susceptibility (single wavy-line) is dressed (double wavy-line) by the nemato-elastic coupling (red crosses). The dashed line is an acoustic phonon. (b) The pairing potential $W_{\mathbf{k},\mathbf{k}'}$ consists of the dressed nematic interaction of strength U , and a non-nematic interaction $V_{\mathbf{k},\mathbf{k}'}$ (gluon-line) of strength V . The former interaction vertex is accompanied by a form factor $f_{\mathbf{k},\mathbf{k}'}$. The interesting regime is $V > U$, i.e., when pairing is dominated by the non-nematic interaction away from the nematic QCP.

neglected.

The effective electron-electron interaction mediated by the nematic variable $\phi_{\mathbf{q}}$ has the form

$$\mathcal{H}_{\text{nem}} = -U \sum_{\mathbf{q}} \chi(\mathbf{q}) \mathcal{N}_{\mathbf{q}} \mathcal{N}_{-\mathbf{q}}, \quad (1)$$

where $\mathcal{N}_{\mathbf{q}} = \sum_{\mathbf{k},\sigma} f_{\mathbf{k},\mathbf{q}} c_{\mathbf{k}+\mathbf{q},\sigma}^\dagger c_{\mathbf{k},\sigma}$, in terms of the creation and annihilation operators of electron with spin σ . The form factor $f_{\mathbf{k},\mathbf{q}} = (h_{\mathbf{k}} + h_{\mathbf{k}+\mathbf{q}})/2$, where $h_{\mathbf{k}}$ transforms as $(k_x^2 - k_y^2)$. The parameter U , with dimension of energy, sets the scale of the nematic interaction. In addition we assume that there is a second interaction $-V^{(s,d)}$ that favors s - and d -wave pairing, respectively, and whose origin is not related to the nematic QCP, such that its strength, as a function of r , stays constant as the system approaches the QCP [10]. Depending on the context, the origin of V can be short wavelength spin/charge fluctuations [5], or it can be due to Mott correlations such as in Anderson's RVB mechanism [3, 4]. Besides its physical relevance for systems such as the FeSC and the cuprates, the inclusion of V allows us to develop a mathematical criteria to distinguish the two limiting cases of “strong” and “weak” enhancement of T_c upon tuning the system to the nematic QCP with $r \rightarrow r_0$.

In the following we solve the linearized BCS equations for superconducting gap $\Delta_{\mathbf{k}}$, assuming singlet pairing. This involves calculating the largest eigenvalue λ satisfying the relation

$$\lambda \Delta_{\mathbf{k}} = \nu_{\text{FS}} \oint_{\text{FS}'} W_{\mathbf{k},\mathbf{k}'} \Delta_{\mathbf{k}'}, \quad (2)$$

where ν_{FS} is the density of states at the Fermi surface, $W_{\mathbf{k},\mathbf{k}'} = V_{\mathbf{k},\mathbf{k}'}^{(s,d)} + U f_{\mathbf{k},\mathbf{k}'}^2 \chi(\mathbf{k} - \mathbf{k}')$ (see Fig. 3(b)), and FS' implies the \mathbf{k}' -integral is restricted to the Fermi surface.

Our goal is to study how $\lambda(r)$ changes as the system is tuned to the QCP, from which we can deduce the variation of $T_c \sim \Lambda e^{-1/\lambda}$, where $\Lambda \ll E_F$ is the high-energy cutoff of the pairing problem. Note, an *important consequence* of the coupling g is that, it is now possible to assume that $T_c(r_0) < T_{\text{FL}}$, the Fermi liquid scale. For $T < T_{\text{FL}}$ the dynamics of the pairing potential is irrelevant, and the problem can be treated within BCS formalism.

In the above model $\lambda(r)$ increases monotonically as the system approaches the QCP, since the nematic interaction itself is attractive and monotonic. However, the crucial question is whether this increment is significant. To address this issue quantitatively, we define $\delta\lambda \equiv \lambda(r = r_0) - \lambda(r = 1)$, and we distinguish between “strong” and “weak” enhancements of T_c , depending on whether $\delta\lambda \gg \lambda(r = 1)$ or not, respectively. Qualitatively, this criteria distinguishes between whether pairing is dominated by long wavelength nematic fluctuations or by a non critical pairing interaction.

Results.— The momentum anisotropy of the susceptibility $\chi(\mathbf{q})$ due to the coupling g can be modeled as follows [28]. (a) For $q_z \leq q_{2d}$, we get $\chi^{-1}(\mathbf{q}) \approx r(\hat{q}) + q_{2d}^2/(2k_F)^2$. The anisotropic mass $r(\hat{q})$ has tetragonal symmetry, and satisfies $r(\hat{q}_{1,2}) = 0$ at the QCP. The simplest function consistent with these requirements is $r(\hat{q}) = (r - r_0) + r_0(q_z/q_{2d})^2 + r_0 \cos^2 2\phi_{\mathbf{q}}$, where $\phi_{\mathbf{q}}$ is the azimuthal angle of \mathbf{q} (see Fig. 2). This region of \mathbf{q} -space also contains the critical modes. (b) For $q_z \geq q_{2d}$, the nemato-elastic coupling can be neglected and $\chi(\mathbf{q}) \approx \chi_0^{-1}(\mathbf{q}) = r + q_{2d}^2/(2k_F)^2$. However, this does not imply singular susceptibility at the QCP, since its location is shifted from $r = 0$ to $r = r_0$. In this region of \mathbf{q} -space the modes are, thus, non-critical.

The main qualitative physics can be already illustrated by considering the simplest case of a single band with a cylindrical Fermi surface around the Brillouin zone center, and where the non-critical pairing term supports s -wave gap with $V_{\mathbf{k},\mathbf{k}'}^{(s)} = V > 0$. The details of the calculation are given in the Supplementary Information (SI). For a uniform gap the leading r -dependence of the eigenvalue $\lambda(r \geq r_0)$ is given by

$$\lambda/\nu_{\text{FS}} = V + \frac{U}{2\sqrt{r}} - \frac{U}{\pi} (\ln \max[r - r_0, r_0] + c_1), \quad (3)$$

where $c_1 = 8/3 - 2 \ln 2 \approx 1.28$. In r.h.s of the above the second term comes from the momentum space (b), as discussed above, where the fluctuations are massive and non-critical. While the third term comes from the region (a) which includes the critical modes. However, since the critical momentum space is rather restricted (equivalently, the critical theory can be mapped to an isotropic model in effective space dimension $d_{\text{eff}} = 5$ [24]), its contribution to the eigenvalue is subleading. Thus, the leading nematic interaction contribution to λ is from the non-critical long-wavelength modes.

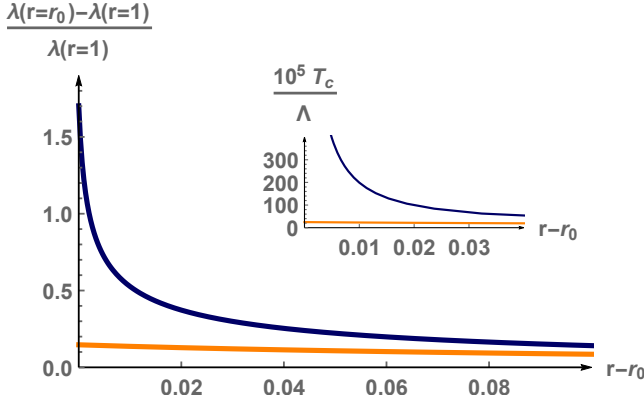


FIG. 4: (color online) Calculated variation of the BCS eigenvalue $\lambda(r)$ (main panel) and the associated superconducting transition temperature $T_c(r)$ (inset) upon tuning the system to the nematic QCP at $r = r_0$. The pairing kernel is shown graphically in Fig. 3. Large enhancement (blue curves) of λ and T_c is observed only when the nemato-elastic coupling strength r_0 is sufficiently weak (see Eq. (4)). Note, in this limit, at the QCP the $T_c \propto e^{-\sqrt{r_0}/(\nu_{FS}U)}$ (not shown) is still a small fraction of the ultraviolet cutoff Λ . In the opposite limit of strong nemato-elastic coupling (orange curves) the enhancement is negligible. This limit is relevant for the Fe-based superconductors such as $\text{FeSe}_{1-x}\text{S}_x$ [17–19].

Importantly, from the above we deduce that, upon tuning the system to the QCP by $r \rightarrow r_0$, the T_c enhancement will be “significant”, i.e., $\delta\lambda \gg \lambda(r=1)$ provided the electron-lattice coupling is weak enough such that

$$r_0 \ll (U/V)^2. \quad (4)$$

In the opposite limit the enhancement is negligible (see Fig. 4). Note, the above condition is trivially satisfied if $U > V$. This implies that the T_c enhancement is strong, irrespective of the strength of the electron-lattice coupling, provided the non-nematic pairing interaction is weak or absent. However, as we argue below, the physically relevant case for the FeSC is $U < V$, i.e., where pairing is dominated by a non-nematic potential sufficiently far from the QCP.

As discussed in the SI, besides the case of the isotropic s -wave gap, we have also considered the following situations. (i) The case of an extended s -wave gap, since the lattice-renormalized nematic interaction is intrinsically anisotropic, and it can give rise to angular variations of the gap. (ii) Motivated by the cuprates, we have considered the case where the non-nematic interaction $V_{\mathbf{k},\mathbf{k}'}^{(d)} = V \cos 2\phi_{\mathbf{k}} \cos 2\phi_{\mathbf{k}'}$ favors a d -wave gap with $\Delta_{\mathbf{k}} = \Delta_0 \cos 2\phi_{\mathbf{k}}$. (iii) Motivated by the FeSC we have considered a system with Fermi pockets at $(0,0)$, $(\pm\pi,0)$ and $(0,\pm\pi)$, with a form of $V_{\mathbf{k},\mathbf{k}'}^{(s)}$ that leads to s_{\pm} -gap. In all these cases we find that qualitatively $\lambda(r)$ is described by Eq. (3), except with different numerical pre-factors.

Thus, the above criterion for T_c enhancement in Eq. (4) is robust.

Discussion.— The strength of the nemato-elastic coupling can be estimated as $r_0 \sim (T_s - T_0)/T_F$, where T_0 is the nominal nematic transition temperature of the electron subsystem in the absence of this coupling (see Fig. 1). Experimentally, T_0 is accessible from, say, electronic Raman scattering [15]. In FeSe we get $T_0 \sim 10$ K, and $T_s \sim 90$ K [31]. We estimate the Fermi temperature from the bottom of the smallest electron pocket as measured by photoemission above T_s , which is around 25 meV in FeSe [17, 32]. Thus, for $\text{FeSe}_{1-x}\text{S}_x$ we estimate $r_0 \sim 0.3$ and $T_{FL} \sim 40$ K. Note, the condition $T_c < T_{FL}$, needed for a weak-coupling theory, is well-respected in this case. A similar estimate for $\text{Ba}(\text{Fe}_{1-x}\text{Co}_x)_2\text{As}_2$ yields $r_0 \sim 0.05$ and $T_{FL} \sim 10$ K [28]. Since, in this system the maximum $T_c \sim 25$ K is comparable to T_{FL} , a more careful quantitative analysis is needed.

The estimation of U and V requires a full microscopic theory of pairing, that is currently unavailable. However, experimentally it is clear that $\text{FeSe}_{1-x}\text{S}_x$ has a nematic QCP around $x \approx 0.16$, but the superconducting T_c remains remarkably flat at this doping [17–19]. This suggests that the condition in Eq. (4) is violated for $\text{FeSe}_{1-x}\text{S}_x$. In turn, this implies that the nemato-elastic coupling is strong enough to suppress the long wavelength nematic fluctuations, and that the pairing interaction is mostly due to a non-nematic origin. A similar case can also be made for $\text{Ba}(\text{Fe}_{1-x}\text{Co}_x)_2\text{As}_2$, where quantum critical nematic fluctuation have been detected only over a narrow doping range of $x = 0.65 - 0.75$ in the low- T superconducting phase [14]. It is remarkable that over the same doping range $T_c(x)$ hardly varies, implying that even here the lattice cutoff is operational. Thus, the dome-like structure of $T_c(x)$ over a wider doping range is likely due to the antiferromagnetic QCP, which is absent in $\text{FeSe}_{1-x}\text{S}_x$.

To summarize, we argued that nemato-elastic coupling can play a crucial role in determining if superconducting T_c is strongly enhanced in the vicinity of a nematic quantum critical point. We showed that, in the presence of a significant non-nematic pairing interaction, strong nemato-elastic coupling implies that the nematic fluctuations have a weak effect on T_c . Based on existing experiments on $\text{FeSe}_{1-x}\text{S}_x$ and on $\text{Ba}(\text{Fe}_{1-x}\text{Co}_x)_2\text{As}_2$ we argued that this is the case of the iron-based superconductors. Most likely, in these materials the main pairing interaction is non-nematic in origin. More generally, from the perspective of material design for high temperature superconductivity, we conclude that (a) hard crystals are better suited for boosting T_c near a nematic quantum critical point, and that (b) the lattice cutoff can be also avoided provided the non-nematic pairing potential is strong enough to guarantee $T_c(r=1) \gg T_{FL}$, in which case the physics of Refs. [7–10] will be operational.

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Supplementary Information for “Pairing instability near a lattice-influenced nematic quantum critical point”

In this appendix we provide the technical details for the calculation of the BCS eigenvalue λ defined in Eq. (2) of the main text which is

$$\lambda \Delta_{\mathbf{k}} = \nu_{\text{FS}} \oint_{\text{FS}'} W_{\mathbf{k}, \mathbf{k}'} \Delta_{\mathbf{k}'}. \quad (\text{S1})$$

The interaction potential is $W_{\mathbf{k}, \mathbf{k}'} = V_{\mathbf{k}, \mathbf{k}'}^{s,d} + U f_{\mathbf{k}, \mathbf{k}'}^2 \chi(\mathbf{k}' - \mathbf{k})$. The form factor $f_{\mathbf{k}, \mathbf{k}'} = (h_{\mathbf{k}} + h_{\mathbf{k}'})/2$, where $h_{\mathbf{k}}$ transforms as $k_x^2 - k_y^2$ in the $k_x - k_y$ plane. A simple choice is $h_{\mathbf{k}} = \cos(2\phi_{\mathbf{k}})$, where $\phi_{\mathbf{k}}$ is the azimuthal angle of \mathbf{k} . Note that the nematic pairing potential is intrinsically anisotropic in the presence of the nemato-elastic coupling. This anisotropy can be taken into account by dividing the momentum space into two regions (a) $q_z \geq q_{2d}$, and (b) $q_z \leq q_{2d}$, and by working with asymptotic forms of χ in these two regions. Therefore, the pairing potential can be broken in three parts

$$W_{\mathbf{k}, \mathbf{k}'} = V_{\mathbf{k}, \mathbf{k}'}^{s,d} + U f_{\mathbf{k}, \mathbf{k}'}^2 \chi(\mathbf{k}' - \mathbf{k})|_{q_z \geq q_{2d}} + U f_{\mathbf{k}, \mathbf{k}'}^2 \chi(\mathbf{k}' - \mathbf{k})|_{q_z \leq q_{2d}} \quad (\text{S2})$$

The asymptotic forms of χ in the two regions are described in the main text. Note, the critical manifold is contained in the third term above, while the second term above involves non-critical modes. Furthermore, we will assume that $V > U$, where V is the strength of the non-nematic pairing potential $V_{\mathbf{k}, \mathbf{k}'}^{s,d}$. Physically this implies that sufficiently far from the nematic QCP the pairing is dominated by the non-nematic term. As noted in the main text, the opposite limit of $U > V$ is trivial, since if the nematic potential already dominates pairing far away from the QCP, then, irrespective of the strength of the nemato-elastic coupling, it will invariably lead to large T_c enhancement because the dominant pairing potential grows (even if it stays finite) as the QCP is approached.

(a) *s-wave superconductivity with a uniform gap and a cylindrical Fermi surface (FS)*. We will assume that the non-nematic pairing potential is a constant with $V_{\mathbf{k}, \mathbf{k}'}^s = V$. Since Eq. (2) of the main text is restricted to the Fermi surface, $q_{2D} = 2k_F |\sin(\frac{\phi_{\mathbf{k}} - \phi_{\mathbf{k}'}}{2})|$ and $\cos^2(2\phi_{\mathbf{q}}) = \cos^2(\phi_{\mathbf{k}} + \phi_{\mathbf{k}'})$. The FS integral turns into angular integrals $\lambda = \langle W_{\mathbf{k}, \mathbf{k}'} \rangle$ where

$$\langle f \rangle = \int_0^{2\pi} \frac{dudv}{(2\pi)^2} f(u, v), \quad (\text{S3})$$

and $u = \phi_{\mathbf{k}} + \phi_{\mathbf{k}'}$ and $v = \phi_{\mathbf{k}} - \phi_{\mathbf{k}'}$. This mean value is to be estimated to lowest order in the parameter $r \leq 1$ which governs the nearness to the QCP (see Fig. 1, main text).

The contribution from the second term of Eq. (S2) is given by

$$U \langle \cos^2 u \cos^2 v \frac{1}{r + (1 - \cos v)/2} \rangle \approx \frac{U}{2\sqrt{r}}.$$

Note, in the above estimation typical $q_z \sim \pi/c$, where c is unit cell length along the z -direction, while typical $q_{2d} \sim r^{1/2}$. Therefore, to leading order in r the constraint $q_z \geq q_{2d}$ is automatically satisfied in the above estimation, even though the angular integrals are performed freely.

The third term of Eq. (S2) involves momentum dependence along the z -direction, and therefore, the estimation of its contribution to $\lambda(r)$ involves averaging along the length of the cylindrical Fermi surface. Anticipating that the typical momentum transfer along z is small compared to Fermi wavevector k_F we can write $\oint_{\text{FS}} \rightarrow \int_0^{2\pi} \frac{d\phi}{2\pi} \int_0^1 d(k_z/k_F)$. This implies that the contribution from the third term of Eq. (S2) is given by

$$2U \langle \cos^2 u \cos^2 v \left| \sin \frac{v}{2} \right| \int_0^1 dx \frac{1}{r - r_0 + r_0 \cos^2 u + \sin^2(v/2) + r_0 x^2} \rangle \approx -\frac{U}{\pi} (\ln \max[r - r_0, r_0] + c_1),$$

where $c_1 = 8/3 - 2 \ln 2 \approx 1.28$. This leads to the equation

$$\lambda(r \geq r_0)/\nu_{\text{FS}} = V + \frac{U}{2\sqrt{r}} - \frac{U}{\pi} (\ln \max[r - r_0, r_0] + c_1), \quad (\text{S4})$$

which is equation (3) of the main text. Note, the leading r -dependence comes from the non-critical modes, rather than the critical ones which have a rather limited volume in momentum-space. The critical contribution gives only to a weak logarithmic dependence which can be ignored to leading order in r .

It is clear from the above that there will be considerable T_c enhancement close to the nematic QCP defined by $r = r_0$ only if in this regime the nematic contribution dominates. This, in turn, is possible only if the nemato-elastic coupling is weak enough such that

$$r_0 < (U/V)^2. \quad (\text{S5})$$

This is the condition mentioned in equation (4) of the main text.

In the following we consider few other cases and we show explicitly that the structure of Eq. (S4) remains the same, only numerical pre-factors change. This implies that the conclusion obtained in Eq. (S5) is robust.

(b) *s-wave superconductivity with higher order gap harmonics.* Keeping s-wave symmetry we can introduce anisotropy in the gap function by considering higher order harmonics as $\Delta(\mathbf{k}) = \Delta_0 + \sqrt{2}\Delta_4 \cos(4\phi_{\mathbf{k}})$. The second term of r.h.s is the normalized first higher order s-wave harmonic. We proceed to project the gap equation onto each orthogonal polynomial to get the secular equation

$$\begin{aligned} \lambda\Delta_0 &= \lambda_{00}\Delta_0 + \lambda_{40}\Delta_4, \\ \lambda\Delta_4 &= \lambda_{40}\Delta_0 + \lambda_{44}\Delta_4, \end{aligned} \quad (\text{S6})$$

where we have defined $\lambda_{nn'} = \langle W_{kk'} g_n g_{n'} \rangle$, with g_n the n-th orthogonal cosine polynomial. The secular system implies that the physical T_c is to be given by the largest value of the matrix $(\lambda_{nn'})$. With the above ansatz for the gap we get

$$\lambda = \frac{\lambda_{00} + \lambda_{44}}{2} + \sqrt{\left(\frac{\lambda_{00} - \lambda_{44}}{2}\right)^2 + \lambda_{40}^2}. \quad (\text{S7})$$

The calculation is then identical to case (a). Ignoring the log corrections from the critical manifold, we find to lowest order in r

$$\lambda(r \geq r_0)/\nu_{FS} = V + \left(1 + \frac{1}{\sqrt{2}}\right) \frac{U}{2\sqrt{r}}, \quad (\text{S8})$$

which is same as in case (a) except for a numerical pre-factor. It is also possible to do the calculation in the limit of an infinite number of s-wave harmonics, and we find that the superconducting eigenvalue goes as $\lambda(r) = V + U/\sqrt{r}$.

(c) *d-wave superconductivity.* Motivated by the cuprates, we take a non-nematic pairing interaction which promotes d-wave superconductivity $V_{\mathbf{k},\mathbf{k}'}^d = 2V \cos(2\phi_{\mathbf{k}}) \cos(2\phi_{\mathbf{k}'})$ on top of the nematic pairing potential. With a d-wave gap ansatz $\Delta(\mathbf{k}) = \Delta_0 \cos(2\phi_{\mathbf{k}})$, we find

$$\lambda(r \geq r_0)/\nu_{FS} = V + \frac{3U}{4\sqrt{r}}. \quad (\text{S9})$$

Thus, once again the BCS eigenvalue is the same as in Eq. (S4) except for a numerical pre-factor.

(d) *The multiband case of Fe-based superconductors.* Motivated by the physics of the Fe-based superconductors, we now consider a three band model with one hole band centered around the (0,0) point of the Brillouin zone and two electron pockets located at $(\pi, 0)$ and $(0, \pi)$ respectively, in the one-Fe/unit cell representation. The non-nematic pairing potential is now a matrix in the band space, and we take it to be

$$V_{\mathbf{k},\mathbf{k}'}^s = -V \begin{pmatrix} 0 & 1/2 & 1/2 \\ 1/2 & 0 & -1/2 \\ 1/2 & -1/2 & 0 \end{pmatrix}. \quad (\text{S10})$$

Note, Eq. (S1) is written with the convention that repulsive interactions have negative sign and attractive ones have positive sign. Thus, the above interaction implies that the non-nematic pairing potential is only inter-band, and that it is repulsive for the electron-hole pairing term, while it is attractive for the electron-electron pairing term. This invariably leads to a s_{\pm} gap, which in the three-band language has the form $\Delta_0(1, -1, -1)$, which is the most discussed gap structure for these systems. Note, the nematic pairing potential is attractive, and is, by definition, intra-band. For circular Fermi surfaces, and assuming that the gaps on each of the pockets are constant, we get, following the case (a)

$$W_{\mathbf{k},\mathbf{k}'} = V \begin{pmatrix} x & -1/2 & -1/2 \\ -1/2 & 2x & 1/2 \\ -1/2 & 1/2 & 2x \end{pmatrix}, \quad (\text{S11})$$

where $x = U/(2\sqrt{r}V)$. This leads to a BCS eigenvalue

$$\lambda(r \geq r_0)/\nu_{FS} = \frac{1}{4} \left(V + \frac{3U}{\sqrt{r}} + \sqrt{9V^2 + \frac{U^2}{r} + \frac{2UV}{\sqrt{r}}} \right). \quad (\text{S12})$$

Since the only energy scales here are V and U/\sqrt{r} , it is simple to check that significant T_c enhancement is only possible if the condition in Eq. (S5) holds. Note also that, while the magnitudes of the gaps become different on the hole and the electron pockets with the inclusion of the nematic pairing, both for small and for large x there is a change in the sign of the gap between the hole and the electron surfaces.

We conclude that in all the above cases the condition for significant T_c enhancement is given by Eq. (S5), while in the opposite limit there is hardly any impact of the QCP on the T_c .